Calculus Review 1st Semester Final Exam

Show All Work on Lined Paper

Since Part I of the test is 28 questions – No Calculator
and Part II of the test is 17 questions – Calculator
This Review has both Non-Calculator and Calculator questions

#1 - #5 on this Review is Non-Calculator
#6-#10 on this Review is Calculator
#11-#15 on this Review is Non-Calculator
#16-#20 on this Review is Calculator

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1. Find equation of the tangent line to the graph $f(x) = x(4-3x)^2$ at $x = 2$

2. $f(x) = \sqrt{4x}$ Find $f'(5)$

3. When is f concave up? $f(x) = \frac{1}{12}x^4 + \frac{1}{2}x^3 - 2x^2 + 5$

4. Find dy/dx at (1, -1) $2x - 3xy - y^3 = 6$

5. $\int_1^3 |x-2|\, dx$

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6. Find Area of the region enclosed by $y = \frac{1}{2}x$ and $y = 2\sin(x)$ in QI only.

7. How many zeros does $f'(x)$ have? $f(x) = x\cos(x)$ $[-2\pi, 2\pi]$

8. Does the graph of $y = \dfrac{-2x}{1-2x}$ have $y = -1$ as an asymptote?

9. Find the Volume of Region R enclosed by $y = -3\cos(x)$ and $y = 0$
   Region R is in the 1st Quadrant. Revolve Region R about y – axis.
   $[0, 2\pi]$

10. Find Average Value of $f(x) = \sqrt{x-1}$ on interval [3.5 , 5.2]

11. At what values of x does f have a relative maximum? $f(x) = \frac{1}{5}x^5 - \frac{13}{3}x^3 + 36x$

12. Let f be function $f(x) = 4x^3$. What are the values of c that satisfy the conclusion of Mean Value Theorem on interval $[-1, 2]$

13. Find dy/dx $f(x) = \dfrac{x^2 - 3}{4x + 5}$

14. In which interval is $f(x)$ decreasing? $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 4x + 6$

15. $\int_0^1 (3x - 1)^4 \, dx$

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16. What is Area of largest rectangle with lower base on x-axis and upper vertices on curve $Y = 4 - x^2$

17. $f(x) = (\cos(2x))(x + 4)^2$ Find $f'(0)$

18. Find the shortest distance from curve $xy = 6$ to the origin

19. $\int_1^4 x(2x - 6)^3 \, dx$

20. $f(x) = x^4 - 5x^3$
$g(x) = 6x^3 + 2x$

$h(x) = f(g(x))$ Find $h'(1.3)$

Calculus Practice #2 – 1st Semester final Exam

Questions 1) to 5) do not use Calculator.

1) Limits: $\lim_{h\to 0}\dfrac{(x+h)^3-x^3}{h}$ at the point x = 2

2) Integration: Calculate $\int \csc^2 x\,dx =$

3) Quotient rule: Find equation of the tangent line to the curve $y=\dfrac{3x+4}{4x-3}$ at the point (1,7).

4) Particle Motion: A particle moves along the x-axis so that at any time t its position is given by $x(t)=\dfrac{1}{2}\sin t+\cos(2t)$. What is the acceleration of the particle at $t=\dfrac{\pi}{2}$?

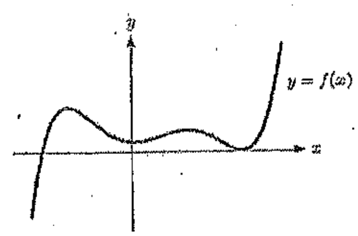
5) Definite Integral: Calculate $\displaystyle\int_0^3 \frac{x}{\sqrt{x^2+16}}dx =$

Questions 6) to 10) use Calculator.

6) Average Rate of Change: Given f(x) = $2x^3$, calculate the average rate of change of f(x) on the interval [0 , 2]

7) Graphic Derivation: Given the graph of the function y = f(x) shown in figure A) on the right, determine the number of times the graph $f'(x)$ will cross the x-axis.

Figure A)



8) Tangent Line: Given the tangent line to the graph of y = sin x at the point $(\dfrac{2\pi}{3},\dfrac{\sqrt{3}}{2})$ determine (with three decimals) at what value of x the tangent line at that point crosses the graph y = sin x.

9) Optimization: A company sells x calculators per week that are sold at a price of (75 – 0.01x) dollars each, and the total cost is ($1850 + 28x - x^2 + 0.001x^3$) dollars. Determine how many calculators the store must sell per week to maximize their net profit each week. Note: Net profit = Total sales minus Total cost.

10) Related Rates: A missile rises vertically from a point on the ground 75,000 feet from a radar station. If the missile is rising at a rate of 16,500 feet per minute at the instant when it is 38,000 feet high, what is the rate of change in radians per minute of the missile's angle of elevation as seen from the radar station?

Questions 11) to 15) do not use Calculator.

11) Implicit Differentiation: Calculate $\dfrac{dy}{dx}$ at the point (2, -3) for the implicit function $y^2 - 2xy = 21$

12) Concavity: What are all the values of x for which the graph of $y = 6x^2 + \frac{x}{2} + 3 + \frac{6}{x}$ is concave downward?

13) Optimization: What is the area of the largest possible rectangle with the lower base on the x-axis, and the two upper vertices on the curve $y = 12 - x^2$?

14) Mean Value Theorem: Given $f(x) = x + \frac{4}{x}$ in the interval [1, 4], find all the values of c in the interval (1, 4) that satisfy the Mean Value Theorem for the given function.

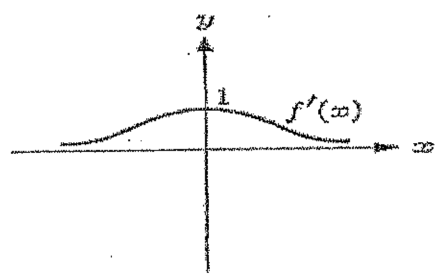
alculus 15) Extrema: If $\frac{dy}{dx} = y\cos x$, and we know that y = 3 when x = 0, then find the function y
3C only

Questions 16) to 20) use Calculator.

16) Particle Motion: A particle moves along the x-axis so that its position at any time t > 0 is given by $x(t) = t^3 - 12t^2 + 36t - 20$ in meters and t in seconds. What is the total distance in meters that the particle travels from t = 2 seconds to t = 9 seconds?

17) Graphing Derivatives: Given $f'(x)$ in Figure A on the right, graph the function f(x).

Figure (A)



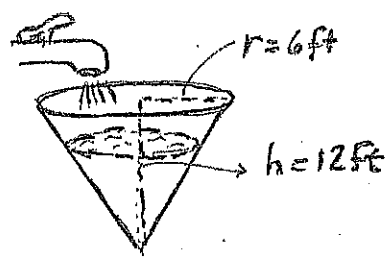
18) Intersection: Given the two functions:

$$\begin{cases} y = -2x^2 + 5x + 3 \\ y = \sqrt{x+5} \end{cases}$$

Determine the coordinates of the two points of intersection with 3 decimal places.

19) Related Rates: The water tank illustrated in Figure B, on the right, has a height of 12 ft and a base of radius 6 ft. If water is poured on the top at a rate of 10 ft^3 per min, at what rate is the water level rising when the depth is 4 ft?

Figure B



20) Inflection Points: Given the function $y = x^3 + x^2 - 5x - 3$, find the following:

a) The coordinates (with 3 decimals) of the relative maximum and relative minimum, if they exist.
b) Find the inflection point or points (with 3 decimals).

AP Calculus Review Chapters 1-3

1. Use the definition of derivative to calculate $f'(-1)$ for $f(x) = 4x^2 - 5x + \pi$. Use this formula: $\lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$
2. Find each of the following limits:

a) $\lim_{x \to -\infty} \frac{5x^2 - 6x^4 - 7x^{13} - 1}{-3x^4 + 4x^{11} + 7}$ b) $\lim_{x \to -\infty} \frac{3x^5 - 2x^7 - 5x - 2\pi}{2x - 3x^6 - 5x^7 - 1}$

c) $\lim_{x \to 25} \frac{\sqrt{x} - 5}{-2x + 50}$

3. Determine the domain of $f(x) = \frac{3x \cdot \sqrt[4]{2x^3 - 5x^2 - 8x + 20}}{-x + 3}$

4. Find the equation(s) of the horizontal asymptotes to $g(x) = \frac{-2x - 5}{7x^2 - 3}$

5. Find the equation(s) of the vertical asymptotes to $g(x) = \frac{3x - 4}{-9x^2 + 16}$

6. Use the definition of derivative to determine that $g(x) = |3 - 2x|$ is not differentiable at x=1.5

7. Find the values of h and k so that f(x) is continuous everywhere. Justify your answer.

$$f(x) = \begin{cases} x^2 - 4xk - 3h & \text{if } x \geq 4 \\ k & \text{if } -2 < x < 4 \\ \frac{3}{2}x^3k - 3xh & \text{if } x \leq -2 \end{cases}$$

8. Find the equation of the normal line to the function $f(x) = \left(\frac{2 - x}{3x - 2}\right)^3$ at the point x=0.

9. The radius of a sphere increased from 20 cm to 20.5 cm. Use differentials to approximate the change in the volume dV.

10. Find $f'(3)$ if $f(x)=h(g(x))$, $g(3)=4$, $g'(3)=2$, $h(4) = -7$ and $h'(4)=5$.

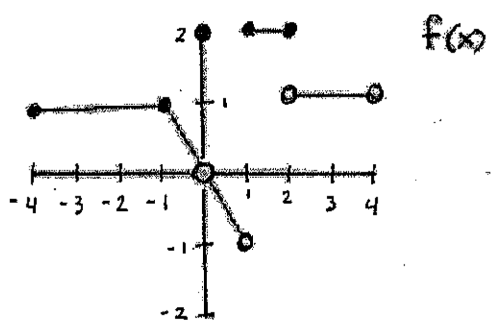
11. Which of the following answers is equivalent to the expression below?

$\lim_{x \to a} \frac{\sin(x) - \sin(a)}{x - a}$ a) 0 b) a c) 1 d) cos(a) e) sin(a)

12. For the graph given at the right, answer the following:

a) $\lim_{x \to 0} f(x)$

b) $\lim_{x \to -1} f(x)$



c) $\lim_{x \to -3^+} f(x)$

d) $\lim_{x \to 1} f(x)$

Calculus AB/AP review

) **Related Rates: Spherical bubble.** Air is pumped into a spherical bubble. When the bubble has a radius of 0.5 cm, the air is flowing into it at a rate of 0.2 cm^3/sec. Find the rate at which the radius of the bubble is changing.

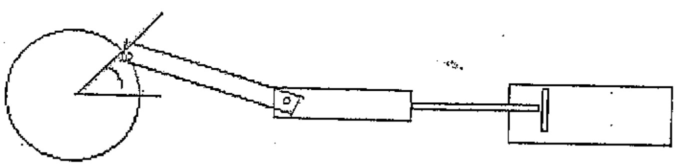
) **Minimize the distance from a curve:** Find the coordinates of a point on the curve $y = \sqrt{x}$ which is closest to the point (5,0).

Maximizing the volume of a box: An open-topped box is to be made from a rectangular piece of material, 8 by 10 inches in size, by cutting equal squares from each corner and turning up the sides. Find the maximum volume enclosed by a box made in this way.

Mean Value Theorem and a parabola: Apply the Theorem to the points at $x = 2$ and $x = 5$ on the parabola $f(x) = x^2 - 6x + 9$

aximize Revenue: Let Revenue, R, depend on the number of units, x, according to the equation. $R = 40x^{3/4} - 3x$.

Related rates: Piston and flywheel:



The piston shown is driving the flywheel. When the crank is in the position shown, the design requires that the flywheel have a speed of 0.4 radians/sec. Find the corresponding speed of the piston. Further data is that the acute angle shown is 30 degrees $(\pi/6)$, and the radius of the crank is 15 cm. (Hint : Use an equation from Polar Coordinates)

)ifferentials: Estimate a root: $\sqrt[5]{32.5}$

Tangent line to a rational function at a given point: Find the equation of the tangent line to $y = \dfrac{x-2}{x+1}$ at the point (1, -1/2).

Compute $\displaystyle\lim_{x \to 5} \frac{x^2 - 2x - 15}{x - 5}$

10). For the following function, what value must be assigned to f(3) to make the function continuous at x=3 ? $f(x) = \begin{cases} x^3 - 20 & \text{for} \quad x > 3 \\ 2x + 1 & \text{for} \quad x < 3 \end{cases}$

11). What is the derivative of $f(x) = \frac{1}{x}\sin(\frac{1}{x})$?

12). Find the equation of the line normal to the tangent of $f(x)=x^2-4x+5$ at $x=1$.

13). Compute dy/dx where $x = 4x^2 y - 2y$.

14). Find the inflection points on the interval $(\frac{-\pi}{2}, \frac{5\pi}{2})$ for the function $f(x) = \sin(x)$.

15). Does $f(x) = \frac{3x^5 - 4x^3 - 2x^2 + 4}{4x^5 - 2x^3 - 4x^2 - 8}$ have a horizontal asymptote? If so, what is its equation?