Calculus Chapter 11 AP Problems

1. Cat # 14
Let S be the series $S = \sum_{n=0}^{\infty} \left(\frac{t}{1+t}\right)^n$, where $t \neq 0$

A) Find the value to which S converges when t = 1.
B) Determine the values of t for which S converges.
C) Find all values of t that make the sum of the series S greater than 10.

2. Cat # 14
Consider the power series $\sum_{n=0}^{\infty} a_n x^n$

where $a_0 = 1$ and $a_n = \left(\frac{7}{n}\right) a_{n-1}$ for $n \geq 1$

A) Find the first 4 terms and the general term of the series.
B) For what values of x does the series converge?
C) If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ find the value of $f'(1)$

3. Cat #14
A) Find the 1st three terms in Taylor series about x=0 for $f(x) = \frac{1}{1-2x}$

B) Find the interval of convergence for the series in Part A
C) Use partial fractions and the result from Part A to find the first five terms in the Taylor series about x=0 for $g(x) = \frac{1}{(1-2x)(1-x)}$

4. Cat #14

Determine all values of x for which the series $\sum_{K=0}^{\infty} \frac{2^K x^K}{\ln(K+2)}$ converges.
Justify

5. Category #14 Let f be the function defined by:

$$f(x) = \frac{1}{1-3x}$$

A) Write the first 4 terms of the Taylor series expansion of f(x)about x=0
B) Find the general term
C) Write the series using correct series notation
D) Using 1st 3 terms of the series, find an approximation of f(-.5)

$$x = -\frac{1}{6}$$

E) Find the value of f at
F) How many terms are adequate for aproximating f(-1/6) with an error not exceeding .02

$$f(x) = \frac{1}{x-1}$$

6. Category #14 Let f be the function defined by

A) Write the 1st 4 terms and general terms of the Taylor Series expansion of f(x) about x=2
B) Use the result from part (a) to find the 1st 4 terms and general term of the series expansion about x=2 for

$$\ln|x-1|$$

C) Use the series in part (b) to compute a number that differs from ln 3/2 by less than .05. Justify.

7. Category #14
A) Show that the series converges for p>1

$$\sum_{n=2}^{\infty} \frac{1}{n^p (\ln n)}$$

B) Determine whether the series converges or diverges for p=1. Show your analysis.
C) Show that the series diverges for

$$0 \le p < 1$$

8. Category #14
Let f be the function given by $f(t) = \frac{4}{1+t^2}$

And G be the function given by $G(x) = \int_0^x f(t)dt$

A) Find the 1st 4 nonzero terms and general term for the power series expansion of f(t) about t=0.
B) Find the 1st 4 nonzero terms and general terms for the power series expansion of G(x) about x=0.
C) Find the interval of convergence of the power series in part (b) (Your solution must include an analysis that justifies your answer)

HP Category = IT

A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x-coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with initial condition $x(0) = -4$.

(a) Find $x(t)$ in terms of t.

(b) Find $\frac{dy}{dt}$ in terms of t.

(c) Find the location of the particle at time $t = 4$.

(d) Find the speed of the particle at time $t = 4$.

(e) Find the total distance traveled from $0 \leq t \leq 3$.