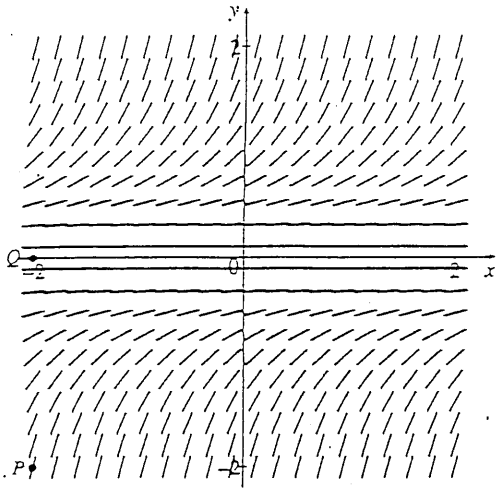
Review for Last Test - AP Problems

Vectors, Euler Tangents, Slope Fields, & Logistic Growth

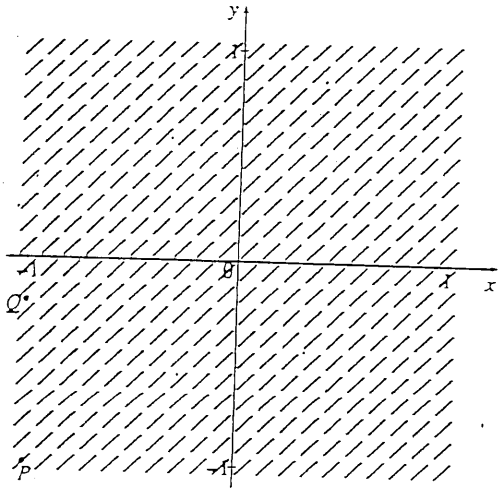
Cat#13 1. A direction field for a differential equation is given below:



(a) Sketch the graphs of the solutions that have initial condition P and initial condition Q.

(b) Examine vertical lines
(c) Examine horizontal lines
(d) Examine 1st Quadrant

Cat#13 2. A direction field for a differential equation is given below:



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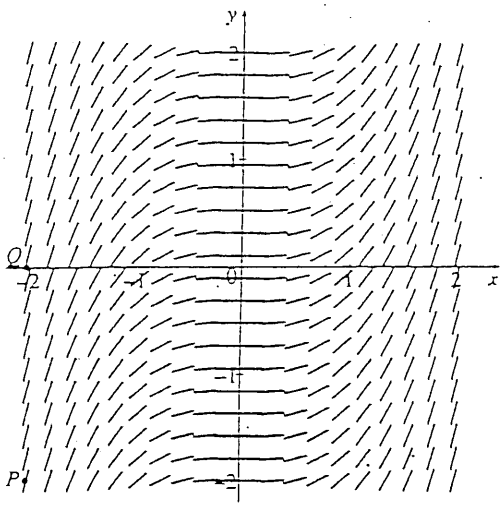
Cat # 13 3. A direction field for a differential equation is given below:



(a) Sketch the graphs of the solutions that have initial condition P and initial condition Q.

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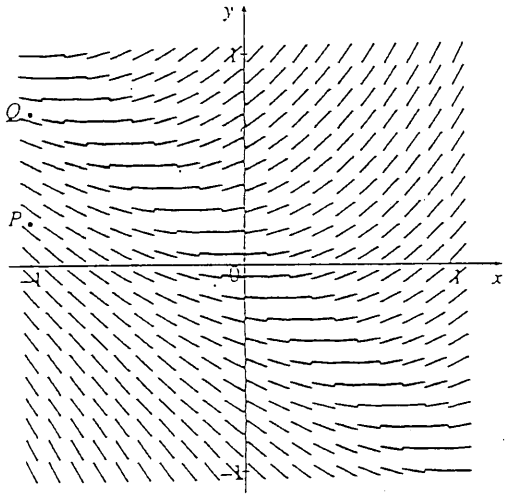
t #13 4. A direction field for a differential equation is given below:



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(c) Examine horiz lines
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VECTOR AND ARC LENGTH

5. A particle moves in the plane so that at any time, t, $0 \le t \le 1$, its position is given

Cat#14 by $x = \frac{1}{4}e^{8t} - 2t$ and $y = e^{4t}$. Let C denote the path traced by the particle.

A) Find the components of the velocity vector for any time t

B) Find the arc length of C

C) Fill in chart and sketch curve. Show orientation

t	x	y
0		
1/4		
1/2		
1		

VECTORS

6. A particle moves around a circle so that its x and y coordiates at time t are $x = 2\cos t$ and $y = 2\sin t$

Cat#14

A) Find the velocity vector

B) Find the acceleration vector

C) Sketch the path of the particle vector $0 \pi \le t \le 2\pi$. Show orientation.

D) Find the velocity and acceleration vectors at $t = \pi/4$

7. A point moves in a plane in such a manner that its coordinates are given in terms of time t by:

Cat#14 $x = 5\cos\frac{\pi}{6}t$

$y = 3\sin\frac{\pi}{6}t$

A) Find magnitude of the velocity of the point at t=2

B) Find the x and y components of the acceleration of the point at t=2

C) Find a single equation in x and y for the path of the point

D) What conic does part © describe?

8. (a) Determine the solution of the differential equation $\frac{dy}{dx} = -y$ where $y(0) = 1$.

Cat#16 (b) Use the solution $y(x)$ from part (a) to calculate $y(0.4)$.

(c) Use Euler's Method with the given step sizes to estimate the value of $y(0.4)$ for the equation given in part (a).

(i) $h = 0.4$

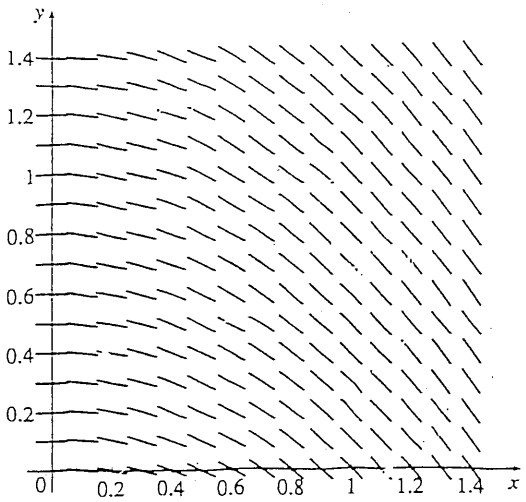
(ii) $h = 0.2$

(iii) $h = 0.1$

(d) Sketch $y(x)$ from part (b) and each of the Euler approximations from part (c) on the same coordinate plane.

9. A direction field for a differential equation is given below. Use a straight edge to draw the graphs of the Euler approximations to the solution curve over the interval $[0, 1.2]$ that passes through $y(0) = 1$. Use as step sizes $h = 1.2$, $h = 0.6$, and $h = 0.2$.

Cat#16



10. Fort Bragg is a town of 5000 people. At 8AM, 500 residents heard a radio announcement about a local political scandal. The rate of growth of the spread of the scandal was jointly proportional to the number of people who heard about the scandal and the number of people who had not heard about it.

Cat #16

A) If at 9 AM, 2000 residents had heard about the scandal, find a mathematical model describing the spread of information.

B) Plot the graph.

C) Estimate how many residents heard about the scandal by 10 AM. Confirm analytically.

D) At what time did half of the population heard about the scandal.

E) Show that by 3 PM, the entire population had heard about the scandal.

AP Category # 14

During the time period $t = 0$ to $t = 6$ seconds, a particle moves along the path given by $r(t) = 3\cos(\pi t)\,\vec{i} + 5\sin(\pi t)\,\vec{j}$

A) Find position of the particle when $t = 2.5$
B) Sketch the path of the particle from $t = 0$ to $t = 6$. Indicate direction.
C) How many times does the particle pass through the point from Part A and at what time, t.
D) Find the velocity vector
E) Find the distance the particle travels from $t = 1.25$ to $t = 1.75$

AP Category # 16

Let y (t) be the velocity in feet per second of a skydiver at time t seconds, $t \geq 0$. After his parachute opens his velocity satisfies the differential equation $\frac{dy}{dt} = -2y - 4$

with initial condition $y(0) = -50$

A) Use separation of variables to find y in terms of t, where t is measured in seconds

B) Draw a slope field of $\frac{dy}{dt} = -2y - 4$

C) Using Euler Tangent Method with initial condition $y(0) = -50$ With step size of t being 0.2. Find $y(0.6)$

D) Terminal velocity is defined as $\lim_{t \to \infty} y(t)$ Find the terminal velocity of the Skydiver to the nearest foot per second.

E) It is safe to land when his speed is 20 ft/sec. At what time t does he reach this speed.

B)