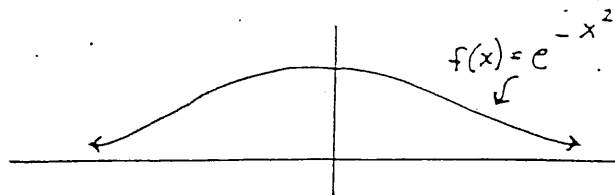


CALCULUS REVIEW CHAPTER 7

Cat #13

1. A) In the interval $0 < x < \frac{\pi}{2}$ find the general solution of the differential equation $(\cot x) \frac{dy}{dx} + y = \csc x$
- B) Find the solution of the differential equation in part (A) that satisfies the condition $y = 0$ when $x = \frac{\pi}{3}$



Cat #7

2. Given the function f defined by $f(x) = e^{-x^2}$
 - A) Find the maximum area of a rectangle that has two vertices on the x-axis and two vertices on the graph of f . Justify your answer.
 - B) Let R be the region in the first quadrant bounded by the x and y axes, the graph of f , and the line $x=k$. Find the volume of the solid generated by revolving R about the y-axis.
 - C) Evaluate the limit of the volume determined in part (B) as k increases without bound.

Cat #6

3. The rate of change in the number of bacteria in a culture is proportional to the number present. In a certain laboratory experiment, a culture had 10,000 bacteria initially, 20,000 bacteria at time t_1 minutes, and 100,000 bacteria at $(t_1 + 10)$ minutes.
 - A) In terms of t only, find the number of bacteria in the culture at any time t minutes, $t \geq 0$
 - B) How many bacteria were there after 20 minutes.
 - C) How many minutes had elapsed when the 20,000 bacteria were observed?

Cat #5

4. A particle moves along the x-axis in such a way that at time $t > 0$ its position coordinate is $x = \sin(e^t)$
 - A) Find the velocity of the particle at time t
 - B) Find the acceleration of the particle at time t
 - C) At what time does the particle first have zero velocity?
 - D) What is the acceleration of the particle at the time determined in part (C)?

Cat # 6

5. Let $y = 2e^{\cos x}$

A) Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

B) If x and y both vary with time in such a way that y increases at a steady rate of 5 units per second, at what rate is x changing when $x = \frac{\pi}{2}$

Cat # 16

6. Given: $5x^3 + 40 = \int_c^x f(t) dt$

A) Find $f(x)$

B) Find the value of c

C) If $F(x) = \int_x^3 \sqrt{1+t^6} dt$, find $F'(x)$

7. Cat # 5 A particle moves along the x -axis so that at any time $t \geq 1$ its acceleration is given by $a(t) = 1/t$. At time $t=1$, the velocity of the particle is $v(1) = -2$ and its position is $x(1) = 4$

A) Find $v(t)$ for $t \geq 1$

B) Find $x(t)$ for $t \geq 1$

C) When did the particle come to rest?

8. Cat # 8 Graph $y = \ln x$ and $y = e^x$

$y = 2 \ln x$ and $y = e^{2x}$

9. Cat # 7 Let f be the function defined by $f(x) = (x^2 + 1)e^{-x}$ for $-4 \leq x \leq 4$

A) For what value of x does f reach its absolute maximum. Justify

B) What is the value of f at its absolute maximum

C) Find the x -coordinates of all points of inflections of f . Justify

$$f'(0) = \lim_{h \rightarrow 0} \frac{e^h - 1}{h}$$

10.

Cat # 4

- A) $f(x) =$
- B) $f'(x) =$
- C) $f'(0) =$
- D) $f'(1) =$

11.

Cat # 10 Let R be the region enclosed by the x-axis, y-axis, $x=2$, and $y = 2e^x + 3x$

- A) Sketch
- B) Find the area of R by setting up and evaluating a definite integral
- C) Find the volume of the solid generated by revolving R about the y-axis (Only set up-don't evaluate)

12.

Cat # 1 Find the equation of the tangent line to the graph of $y = xe^{1/x^3} + \ln |2 - x^2|$ at the point P (1,e)

13.

Cat # 12 Suppose $f(x) = e^{2x} + 2e^x + 1$, where $x \geq 0$

- A) Prove that f has an inverse function f^{-1} and state its domain.
- B) Find $f^{-1}(x)$ and $D f^{-1}(x)$
- C) Find the slope of the tangent line to the graph of f at the point (0,4) and the slope of the tangent line to the graph of f^{-1} at (4,0)

14.

Cat # 11 Find the volume of the region bounded by the graphs

of $y = e^{4x}$, $x = -2$, $x = -3$, $y = 0$ revolved about the x-axis.

15.

Cat # 16 Radium decays exponentially and has a half-life of approximately 1600 years -

- A) Find a formula for the amount $q(t)$ remaining from 50 milligrams
- B) When will there be 20 mg left?

16. Cat # 11 Region R is bounded by the graphs of $xy = 1$, $x = 1$, $x = 2$, and $y = 0$.

- A) Find the area of R
- B) Find the volume of the solid figure generated by revolving the region R about the x-axis.
- C) Find the volume of the solid figure generated by revolving the region R about the line $x = 3$.

17. Cat # 16 The radioactive element carbon 14 has a half-life of 5750 years. If 100 grams of this element are present initially, how much will be left after 1000 years?

18. Cat # 16 According to United Nations data, the world population at the beginning of 1975 was approximately 4 billion and growing at the rate of 2 % per year.

- A) Estimate the world population at the beginning of the year 2000.
- B) In how many years would the population be doubled?

#19 One More Review Problem - Ch 7

$$\frac{dy}{dx} = (3-2x)y^3$$

A) Find $\frac{d^2y}{dx^2}$

B) Evaluate $\frac{d^2y}{dx^2}$ at Point (2,1).

c) Solve the Differential Equation
Using separation of variables

$$\frac{dy}{dx} = (3-2x)y^3$$